Development and research of the algorithm for determining the maximum flow at distribution in the network

Abstract: An upgraded version of Ford-Fulkerson algorithm is proposed, which allows to determine the maximum flow and the distribution flow in branches of the network. Results of simulation using this algorithm are given. They show its effectiveness in comparison with the other variants of flow distribution in the network at change of the capacity of branches randomly in time.

Keywords: Flows of cars, Ford-Fulkerson algorithm, Ford-Fulkerson theorem, maximum flow, algorithm of reducing transit flows.

1 Introduction

In order to organize rationally the traffic flow of vehicles, it is necessary to evaluate the maximum flow in the network, to find the most effective distribution of the flow, to identify bottlenecks and eliminate them in a timely manner. Simultaneously with these tasks, it is necessary to evaluate the total cost of time expenditure for vehicles as they move from the starting point to the destination.

The following researchers devoted their works to issues of finding an integral maximum flow of metropolitan transport network: Zhogal S.I., Maksimey I.V. [1], Zaichenko Yu.P. [2], Polyakov K.Yu. [3], Sobol I.M. [4], Sukach E.I. [5] and others. The analysis of a number of literary sources for finding the maximum flow [1-6], the analytical models of research of operations [7] and the automatic control theory [9] allowed to justify the possibility of using simulation to study the dynamics of traffic flows in the region [5]. Based on this analysis, the relevance of developing an algorithm of flows superposition in accordance with Ford-Fulkerson theorem was revealed, which allows to determine the maximum flow for the integrated area of the regional network.

Today, there is an extensive literature on the study and modeling of road traffic flows. Several academic journals devoted exclusively to the dynamics of traffic. The largest are the Transportation Research, Transportation Science, Mathematical Computer Simulation, Operation Research, Automatica, Physical Review E, Physical Reports. The number of articles published in the hundreds.

The development and research on the effectiveness of various methods of traffic control (TC), the laws of their behavior on the road network (RNW) are devoted to D. Drew [1] H. Enos, T. Hamada [2], Gaishun [3] Thomas H. Kormen [4]. In recent decades, the Russian traffic management practice on the road network of the city has accumulated considerable experience, scientific and methodological foundations of which are summarized in the works V.V. Zyryanov, V.T. Kapitanov [5], G.I. Klinkovshryn [6], Yu. A. Kremenets, M.P. Pecherski, M.V. Yasheva etc.

Issues finding an integral maximum flow metropolitan transport network devoted to the work of researchers: Zhogal S.I., Maksimov I.V. [7], Zaichenko Y.P. [8], Polyakov K.Y. [9], Shvetsov V.I. [10], Sukach E.I. [11], and others. The analysis of a number of literary sources for finding the maximum flow [1-6], the analytical models of operations research [7,8,12] and the automatic control theory [9] allowed to justify the possibility of using simulation to study the dynamics of traffic flows region [10,11]. On the basis of this analysis has revealed the relevance of developing a superposition algorithm flows in accordance with the Ford-Fulkerson theorem, which allows to determine the maximum flow for the integrated area of the regional network.
2 Solution of the problem of finding the maximum flow in the transport network using Ford-Fulkerson algorithm

Using the algorithm for finding the maximum flow at the initial stage. Let $G = (N, A)$ be an oriented network with one source $s \in N$ and one sink $t \in N$, and let the edges $(i, j) \in A$ have a limited capacity. The problem of maximal flow is to find the flow on edges, which belong to the set $A$, so that the resulting flow moving from the source $s$ to the sink $t$, is maximal. It is assumed that the source can act as unlimited flow, the condition of flow conservation is fulfilled for each intermediate node of the network and capacity $U_{ij}$ of each edge is a finite upper limit of the flow $f_{ij}$ along this edge.

Maximum flow problem could be represented as the following tasks: maximize

$$\sum_{i \in N} f_{in}$$  \hspace{1cm} (1)

subject to

$$\sum_{j} f_{ij} - \sum_{j} f_{ji} = 0, \quad i \neq 1, i \neq n,$$  \hspace{1cm} (2)

$$f_{ij} \leq U_{ij}, \quad (i, j) \in A.$$  \hspace{1cm} (3)

To solve this problem, it is possible to use the usual simplex method. However, there is a more effective procedure for finding a solution to this problem. The algorithm starts with a feasible solution. Then, the procedure of marks distribution is done, which is developed by Ford and Fulkerson [4]. This algorithm helps to determine other admissible flow of greater value. In the given algorithm, the nodes are treated as waypoints of flow routing, and the edge is as distribution channels. For a formal description of the algorithm, it is necessary to induce two basic concepts - mapping and augmenting flow path [4].

Mapping the node is used to specify both values of the flow and the flow source that alters the current quantity of flow along the edge connecting the source with the considered node. If flow units $q_j$ are sent from node $i$ to node $j$, and causes an increase in flow along this edge, the node $j$ is mapped from node $i$ by symbol $+q_j$.

In this case, the mark $[+q_j, i]$ is assigned to the node $j$. Similarly, if sending $q_j$ flow units reduces the flow in the edge, then the node $j$ is marked from the node $i$ by the symbol $-q_j$. In this case, the mark $[-q_j, i]$ is assigned to the node $j$.

Current flow from node $i$ to node $j$ is increased when additional $q_j$ units of flow are sent to the node $j$ along the oriented edge $(i, j)$ in a direction similar to its orientation. In this case, an edge $(i, j)$ is called direct.

Current flow from $i$ to $j$ is reduced when $q_j$ units of the flow are sent to the node $j$ over the oriented edge $(i, j)$ in the direction opposite to its orientation. In this case, the edge $(i, j)$ is called direct.

If the node $j$ is mapped from the node $i$ and the edge $(i, j)$ is direct one, the edge on this flow increases and the value corresponding to the remaining unused capacity of the edge should be adjusted properly. This quantity is called the residual capacity of the edge. If a mark is assigned to some node, and it uses a direct branch, it can have only positive "residual capacity". Besides, node $j$ can be marked from node $i$ only after assigning a mark to the node.

Augmenting path of flow from $s$ to $t$ is defined as a linked sequence of direct and inverse edges, along which it is possible to send several flow units from $s$ to $t$. Flow along each direct edge is increased, without exceeding its capacity, and flow on each reverse edge is reduced while remaining non-negative.

Augmenting path of flow is used to select such method of flow altering where the flow in the node $t$ is increased and at the same time, condition of flow conservation for each internal node of the network is not violated.

3 The procedure for placing marks for the problem of maximum flow

Maximum flow problem is common in practice, and the number of nodes and edges in the network often gets to several thousands. Therefore, to solve such problems it is necessary to use effective computation procedure. Due to the simplicity of setting the problem of maximum flow, the efficient recurring algorithm was developed for search of optimum solution (maximum flow) using procedure of placing marks. We shall now describe this algorithm.

Let $(i, j)$ - oriented edge leading from node $i$ to node $j$. The flow through it can be increased on $q_j$ units if an edge $(i, j)$ is straight and mark $[+q_j, i]$ is assigned to the node. Suppose that the flow $f_{ij} \geq 0$ is $f_{ij} \leq U_{ij}$ is
already assigned to the edge \((i, j)\). It is obvious that the value \(q_j\) cannot exceed the residual capacity \(U_{ij} - f_{ij}\). But this is not enough to mark the node \(j\) because it is not always possible to get \(U_{ij} - f_{ij}\) flow units from the node \(i\). The number of flow units that can be sent to the node \(j\) can be sent is the same as the number of units added to node \(i\), so the maximum is \(q_i\). Consequently, the flow along the straight edge \((i, j)\) can be increased by the amount on quantity \(q_j\), where \(q_j = \min\{q_i, U_{ij} - f_{ij}\}\).

Similarly, you can mark node \(j\) if the edge \((j, i)\) is a reverse one. Reducing the flow along the edge \((j, i)\) is only possible when \(f_{ji} > 0\). This flow can be reduced maximally only on the number of units in the flow, which can be taken from node \(i\), so on the quantity \(q_i\). Consequently, the reverse flow of the edge \((j, i)\) can be reduced by \(q_j\), where \(q_j = \min\{q_i, f_{ji}\}\).

The algorithm of placing marks functions in the following way. Initially, a mark \([\infty, -]\) is assigned to the source, which indicates that flow of infinitely large value can flow from this node. Further one looks for the augmenting path of flow from the source to the sink passing the marked nodes. All nodes, other than the source, are not marked at the initial moment. It is necessary to pass to the sink along the straight and reverse edges and consistently mark their nodes. There are the following two cases:

1. The mark \([+q_i, k]\) is assigned to the sink. In this case, the augmenting path of flow is found, and it can be increased or decreased on the value \(q_i\) along each edge of the path.

2. Sink \(t\) can not be marked. This means that augmenting path of flow cannot be found, therefore, the created edge flows form an optimal solution (maximum flow).

To illustrate the algorithm, consider finding the integral flow section of the regional transport network, represented as a graph in Figure 1.

The set input to the network is represented by peaks 1 and 2. The set of output nodes is set 9 and 11. The following streams are discussed in transit network \((1, 2, 9, 11)\), \((1, 11)\), \((2, 9, 2, 11)\), respectively, four matrix efficiencies \((1_9, 1_{11}, 2_9, 2_{11})\) and appropriate distribution of value streams over the network branches \((X^{1-9}, X^{1-11}, X^{2-9}, X^{2-11})\).

To generate a list of the most effective streams reject the least efficient flows in such a way so as not to leave any entrance without at least one of the effluent and no way out without at least one of the incoming stream. For this example proved to be the most effective streams flows \(1 \rightarrow 9\) with 41 units of flow quantity, \(1 \rightarrow 11\) to 43 units and the quantity of flow \(2 \rightarrow 9\) with 34 units of the quantity of flow.

Summing up the matrix of maximum flows \(|X^{1-9}|, |X^{1-11}|, |X^{2-9}|, \) and obtain the matrix integral transit flow \(\Sigma X\). By subtracting the bitmap matrix \(|C - \Sigma X|\) from the matrix of network bandwidth \(|C|\) we obtain the matrix \(|C - \Sigma X|\), which has the following form:

\[
\begin{bmatrix}
0 & 5 & -19 & -6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -3 & -17 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 17 & 0 & -20 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -15 & 4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 16 & 0 & 0 & 3 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & -18 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Negative values of the elements of the matrix shows the lack of capacity of the relevant branches of the network in the event of traffic through the network at the same time all posted flows. Choose a branch flow of the sum on which most exceeds its capacity. For this example, this branch \((3.6)\) - the smallest element of the matrix \(|C - \Sigma X|\).

Select all the way, which was saturated with the branch \((3.6)\) in the course of solving the problem of maximum flow for each of the transit areas and the value of delta, which increases the flow along these paths.

For transit directions \(1 \rightarrow 9\) it will be the way:

- \((1, 3) \rightarrow (3, 6) \rightarrow (6, 9), \Delta = 16;\)
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- (1,3) → (3,6) → (6,10) → (10,9), Δ = 4.

For transit directions 1 → 11:
- (1,3) → (3,6) → (6,10) → (10,11), Δ = 20.

The transit direction 2 → 9, none of the paths in the course of solving the maximum flow problem did not pass through the vertex (3,6). To the total flow could go through branch (3.6), reduces the flow of transit directions in selected ways so that the entire flow of the sum decreased by 20 units, with each flow path is reduced in proportion to the value of Δ. For transit directions 1 → 9 by the flow path 1 decrease by 8 units, and flow along the path 2 is reduced by 2 units. For transit directions 1 → 11 flow along the path 1 decreases by 10 units. To this end, we take away from the matrix $I_{11} - I_{19}$ elements, corresponding to the nodes of ways the amount by which reduces the flow on the road.

Next translated values of the elements of the matrix. The result is a new matrix in which the element (3,6) is 0 since flow through the branch network (3.6) has been reduced.

The process of reducing the flow under consideration is repeated as long as negative elements in the matrix $|C - \Sigma X|$ will be. When all elements of the matrix $|C - \Sigma X|$ will be non-negative, it means that the network throughput capacity enough branches to all reduced transit flows could exist simultaneously in the network. In this case, the algorithm ends reduce transit flows and solve the problem of maximum flow is the total matrix $|\Sigma X|$.

The following matrix has been received for consideration by the count of consecutive decreases $|\Sigma X|$:

$$
\begin{bmatrix}
0 & 12 & 20 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 10 & 17 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 14 & 13 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 20 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

The flux after reducing transit 1 → 9 was 24 units, transit 1 → 11 - 25 units, transit 2 → 9 - 15 units. The total value of the three streams amounted to 64 units.
The essence of the theorem on the maximum flow and minimum cut is that maximum flow in a network with limited capacity can be found by calculating the total capacity of all cuts and choosing the minimum one among the obtained values. In solving the maximum flow problem, this result is of little practical value because it does not give any information about flows along edges themselves. However, this result is important from a theoretical point of view and is used often at development of complex flow algorithms or analysis of solutions for optimality.

Ford-Fulkerson algorithm has some limitations, which are not fulfilled in practice:

First, the capacity of the network of branches should be a non-negative integer determined numbers. The capacity is not constant in the real network and depends on the probability of such factors as congestion, road conditions, the parameters of the environment. Congestion on the various cuts of the road is different and depends on the availability of internal traffic flow at the site, which may be considered as interference for the movement of the transport unit from the initial network point to the final one. Road condition is determined by its deterioration, operating conditions, the influence of weather conditions. The parameters of the environment vary depending on the season and time of day. As a rule, these factors are interconnected in the transport network.

Second, it is proposed to start seeding Ford-Fulkerson algorithm with some initial flow already existing in the network. The choice of the initial flow does not change the value of the maximum flow but the distribution of the maximum flow in the network.

Third, the problem of finding the maximum flow is solved for a single flow through the network, i.e. there should be only one entrance in the network itself - the network node through which vehicles fall into the network, and only one way out - the network node through which they quit the network.

Based on the above, as a way out it is proposed to use simulation of traffic in the network, which allows to take into account a large number of external factors, and to link the programmed model to its real representation as close as possible. The method allows to determine the maximum flow and the distribution of branches in the network, which is the most effective among the other options of distribution of flows in the network, where the capacity of branches change randomly over time.

Here is an example of using the algorithm of Ford-Fulkerson. Starting transport network is shown in Figure 3.

Assume that the implemented algorithm, giving preference to increasing the maximum length of the chains. In this case, in the first step we will allow additional flow through the circuit (A, B), (B, C), (C, D).

In the second step, select circuit (A, C), (C, B), (B, D). Since the arc (C, B) is not consistent, it started up on the value stream will be deducted from the value stream obtained in the previous step. We received net (Fig.4) is almost equivalent to the original.

It is clear that in order to find the maximum flow needed in 1000 iterations. At that time, as if we have chosen in the first step the chain (A, B), (B, D), then the result would be obtained in a single iteration. In practice, the amount of bandwidth often depends on the unit, and can be of great importance.

By adding flow-enhancing way to the existing flow, maximum flow is obtained when it is impossible to find a way of increasing. However, if the amount of bandwidth - irrational number, the algorithm can operate indefinitely. The integers such problems do not arise, and the work is restricted $O (E * f)$, where $E$ - the number of vertices in the graph, $f$ - the maximum flow in the column, as each increasing path can be found for $O (E)$ and increases the flux as at least 1.
5 Conclusion

1. The relevance of developing the flows superposition algorithm was identified in accordance with the Ford-Fulkerson theorem, which allows to determine the maximum flow for the integrated area of the regional network.

2. The approach was proposed in the problem of finding the maximum flow in the transport network at the initial stage of Ford-Fulkerson algorithm.

3. Due to simplicity of setting the problem of maximum flow, an efficient recurring algorithm for finding optimal solutions (maximum flow) was developed using the procedure of placing marks.

4. The Ford-Fulkerson theorem of maximum flow and minimum cut was proposed and proved.

5. Possibility of simulation of traffic in the network is shown, which allows to take into account a large number of external factors, and to link the programed model to its real representation as close as possible. The method is the most effective one among the other options for flow distribution in the network, where the capacity of branches vary in time randomly.

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References
